FIT and MTBF for Non-Repairable Systems

Dr. Horst Lewitschnig

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Reliability Key Figures

FIT-Rate

The FIT-Rate is the number of expected failures per 10^9 operating hours.

FIT-Rate - alternative formulations

- The FIT-Rate is the average ppm-value per 10^3 operating hours.
- FIT is the λ of the exponential distribution.

MTBF vs. FIT-Rate

$$MTBF = \frac{10^9}{FIT} \text{ hours}$$

FIT-Rate for Exponential Distribution

Typically, the FIT-Rate refers to the cumulative operating hours.

Fails	Devices	operating hours / device	operating hours overall	FIT-Rate
1	1	10^9	10^9	1
1	10^3	10^6	10^9	1
1	10^6	10^3	10^9	1
1	10^9	1	10^9	1

MTBF for Non-Repairable Systems

Assumption: Exponential distribution, $F(t) = 1 - e^{-\lambda t}$.
Order statistics

$$F_{t_{(i)}}(t) = \sum_{j=i}^{n} F(t)^j [1 - F(t)]^{n-j}, \quad 1 \leqslant i \leqslant n.$$

Time to first failure

$$P(min(T_1, T_2, \ldots, T_n) < t).$$

Distribution of the minimum

$$F_{t_{(1)}}(t) = 1 - [1 - F(t)]^n = 1 - e^{-n\lambda t},$$

$$E_{t_{(1)}} = \frac{1}{n \cdot \lambda}.$$

MTBF for Non-Repairable Systems

In operating hours:

$$n \cdot E_{t_{(1)}} = \frac{n}{n \cdot \lambda} = \frac{1}{\lambda}.$$

Memoryless of the exponential distribution:

$$P(T > t + s | T > s) = R(t|s) = \frac{R(t+s)}{R(t)} = \frac{e^{-\lambda(t+s)}}{e^{-\lambda(s)}} = e^{-\lambda(t)},$$

$$F(t|s) = F(t).$$

MTBF for Non-Repairable Systems

After the 1st failure, $n-1$ devices remain. Time to the next failure:

$$P(min(T_1, T_2, \dots, T_{n-1}) < t).$$

Distribution of the minimum

$$F_{t_{(1)}}(t) = 1 - [1 - F(t)]^{n-1} = 1 - e^{-(n-1)\lambda t},$$

$$E_{t_{(1)}} = \frac{1}{(n-1) \cdot \lambda}.$$

In operating hours:

$$(n-1) \cdot E_{t_{(1)}} = \frac{n-1}{(n-1) \cdot \lambda} = \frac{1}{\lambda}.$$

Reliability Key Figures

FIT-Rate for Exponential Distribution

Typically, the FIT-Rate refers to the cumulative operating hours.

	time	in operating hours
Expected time until 1st failure	$\frac{1}{n \cdot \lambda}$	$\frac{n-1}{(n-1) \cdot \lambda} = \frac{1}{\lambda}$
Expected time from 1st to 2nd failure	$\frac{1}{(n-1) \cdot \lambda}$	$\frac{n-1}{(n-1) \cdot \lambda} = \frac{1}{\lambda}$
Expected time from 2st to 3rd failure	$\frac{1}{(n-2) \cdot \lambda}$	$\frac{n-2}{(n-2) \cdot \lambda} = \frac{1}{\lambda}$
...	...	$\frac{1}{\lambda}$

$\Rightarrow$ constant failure rate.