

# FIT and MTBF for Non-Repairable Systems

Dr. Horst Lewitschnig

2025-01-10

## FIT-Rate

The FIT-Rate is the number of expected failures per  $10^9$  operating hours.

## FIT-Rate - alternative formulations

- The FIT-Rate is the average ppm-value per  $10^3$  operating hours.
- FIT is the  $\lambda$  of the exponential distribution.

## MTBF vs. FIT-Rate

$$MTBF = \frac{10^9}{FIT} \text{ hours}$$

## FIT-Rate for Exponential Distribution

Typically, the FIT-Rate refers to the cumulative operating hours.

Fails	Devices	operating hours / device	operating hours overall	FIT-Rate
1	1	$10^9$	$10^9$	1
1	$10^3$	$10^6$	$10^9$	1
1	$10^6$	$10^3$	$10^9$	1
1	$10^9$	1	$10^9$	1

## MTBF for Non-Repairable Systems

Assumption: Exponential distribution,  $F(t) = 1 - e^{-\lambda t}$ .

Order statistics

$$F_{t_{(i)}}(t) = \sum_{j=i}^n F(t)^j [1 - F(t)]^{n-j}, \quad 1 \leq i \leq n.$$

Time to first failure

$$P(\min(T_1, T_2, \dots, T_n) \leq t).$$

Distribution of the minimum

$$F_{t_{(1)}}(t) = 1 - [1 - F(t)]^n = 1 - e^{-n\lambda t},$$
$$E_{t_{(1)}} = \frac{1}{n \cdot \lambda}.$$

## MTBF for Non-Repairable Systems

In operating hours:

$$n \cdot E_{t(1)} = \frac{n}{n \cdot \lambda} = \frac{1}{\lambda}.$$

Memoryless of the exponential distribution:

$$P(T > t + s | T > s) = R(t|s) = \frac{R(t + s)}{R(s)} = \frac{e^{-\lambda(t+s)}}{e^{-\lambda(s)}} = e^{-\lambda(t)},$$
$$F(t|s) = F(t).$$

## MTBF for Non-Repairable Systems

After the 1st failure,  $n - 1$  devices remain. Time to the next failure:

$$P(\min(T_1, T_2, \dots, T_{n-1}) \leq t).$$

Distribution of the minimum

$$F_{t(1)}(t) = 1 - [1 - F(t)]^{n-1} = 1 - e^{-(n-1)\lambda t},$$

$$E_{t(1)} = \frac{1}{(n-1) \cdot \lambda}.$$

In operating hours:

$$(n-1) \cdot E_{t(1)} = \frac{n-1}{(n-1) \cdot \lambda} = \frac{1}{\lambda}.$$

## FIT-Rate for Exponential Distribution

Typically, the FIT-Rate refers to the cumulative operating hours.

	time	in operating hours
Expected time until 1st failure	$\frac{1}{n \cdot \lambda}$	$\frac{n}{n \cdot \lambda} = \frac{1}{\lambda}$
Expected time from 1st to 2nd failure	$\frac{1}{(n-1) \cdot \lambda}$	$\frac{n-1}{(n-1) \cdot \lambda} = \frac{1}{\lambda}$
Expected time from 2nd to 3rd failure	$\frac{1}{(n-2) \cdot \lambda}$	$\frac{n-2}{(n-2) \cdot \lambda} = \frac{1}{\lambda}$
...	...	$\frac{1}{\lambda}$

⇒ constant failure rate.